Projects with Uncertain Requirements and Deadlines

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Abstract

We analyze a dynamic moral hazard problem in teams with imperfect monitoring in continuous time. In the model, players work together to achieve a breakthrough in a project while facing a deadline. The effort needed to achieve a breakthrough is unknown, but players have a common prior about its distribution. This makes the model very flexible since the distribution over the required effort for a breakthrough can model different types of projects.

We characterize the equilibrium and the welfare-maximizing effort path for general distributions of this breakthrough effort and show that three effects are at work: free-riding (i.e., working less), delay of effort (i.e., working later), and an encouragement effect (i.e., working more if others worked more in the past). This encouragement effect increases or decreases the amount of work players put into the project, depending on the type of uncertainty faced.

Keywords: Moral hazard in teams, public good provision, procrastination, projects, strategic experimentation

JEL codes: C72, C73, D81, D82, H41

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1. Introduction

This paper analyzes a dynamic moral hazard problem in teams who work on a project with imperfect monitoring in continuous time. In addition to a deadline, the team faces uncertain objectives, i.e., they don't know how much work is needed to complete the project.

Working in projects, i.e., working together towards a fixed goal after which your team will be terminated, is not only the most common way of working in consulting firms but is getting more and more common in most workplaces (Harvard Business School Press (2004)) and even in the classroom (Hutchinson (2001)).

In this paper, projects have uncertain requirements or changing objectives, which are, unfortunately, quite common.[1]

We show how different types of uncertainty in a project's requirements affect the players' behavior.

The main features of this model are:

- **public benefits**, which are realized upon completion of a project in the form of a lump-sum payment,
- **private costs**, which are assumed to be quadratic,
- an **unknown threshold for success** or uncertain requirements, with a commonly known distribution, which we will call *breakthrough-effort distribution*,
- **unobservable efforts**, so only the player's own effort is known,
- and a **deadline**, after which the team can not complete the project anymore.

In the model, the players exert effort over time until the deadline is reached or the project is successful. While doing so, they only know that they have not been successful yet. A project is successful when an unknown breakthrough effort threshold is reached, which is perceived as random by the players. Projects in this model are described by the assumed distribution of the breakthrough effort. This *breakthrough-effort distribution* can cover many different projects, e.g., projects in which only the current effort influences the probability of success or projects during which players learn about the quality of the project while trying to complete it. One simple example of a breakthrough-effort distribution is the uniform distribution on $[\underline{e}, \overline{e}]$. This means that the players think that the project needs effort between

[1] According to a survey by Taylor (2000) unclear objectives and requirements are the most common cause for failure of IT projects.

$\underline{e}$ and $\overline{e}$ to be completed. For examples of different types of projects and the corresponding breakthrough-effort distributions, see Appendix C.

While this breakthrough-effort distribution can cover many different types of uncertain requirements, the uncertainty is always about the amount of effort players have to exert to succeed. As everyone knows that they have been successful as soon as enough effort was spent, uncertainty is resolved entirely at that point. Thus, there is never uncertainty if enough effort has already been provided (i.e., the question of "Have I studied enough to pass the exam?" can not be answered by this model) and hence no over-provision of effort.

We find that there are three different effects at work in the equilibrium: **free-riding**, which reduces the overall effort the more players are working on the project. The second effect is **encouragement**, which depends on the threshold distribution: Given a decreasing hazard rate, my own effort encourages the other players to work less, while given an increasing hazard rate, my work encourages my coworkers to work more in the future.[2] The last effect is **delay of effort**, which causes players to work later rather than earlier, even with the presence of a discount rate, which lets players want to have a breakthrough as soon as possible.[3] The encouragement effect in this model allows us, due to the very general project structure, to show the influence of project types on strategic behavior. We observe that certain types of projects lead to a positive encouragement effect, i.e., that my work encourages others to work more. In contrast, other types of projects lead to a negative encouragement effect. This positive encouragement effect is very similar to strategic complementarity and the negative encouragement effect to strategic substitutability.

Delay of effort as a result of rational players has not been analyzed before in this context. In this model, the delay is caused by convex costs, a deadline, and uncertainty about the effort required for a breakthrough. It causes players to shift effort towards the end, despite having a strong incentive to finish the project early. While it is not a strategic effect, it gives insights into a team's optimal behavior when there is a deadline.

This paper's main contribution is that we allow for unknown effort requirements, which are not limited to a particular form, thus allowing us to describe different projects just using the prior about the requirements.

[2] The hazard rate can be described as the effect of past effort on the effectiveness of the current effort.

[3] This effect is more than just a consequence of discounting, as the discount rate does affect not only the costs but also the benefits. As the benefits are, by design, later than the costs and higher than the expected costs, a discount rate lets players work earlier, not later.

Our results also have implications on the evaluation of projects: As we observe a last-minute rush even in the equilibrium effort as well as in the welfare-maximizing solution, we know that delaying effort might not only be a rational consequence of unclear objectives but welfare-maximizing.
Furthermore, while some classes of uncertain objectives have a discouragement effect and lower the overall effort workers spent, others have an encouragement effect and might help complete a project that would otherwise not have been completed.

The paper is organized as follows: In the next section, I will give an overview of the relevant literature and how this work fits into it. In Section 3, I will explain the model. In Section 4, I derive the optimal effort for the non-cooperative and the welfare-maximizing case and show that three effects are at work: free-riding, encouragement, and delay of effort. Finally, I will give some concluding remarks in Section 5.

2. Comparison to the Literature

This paper is related to different fields of the literature: Holmstrom (1982) started the game-theoretic literature on *moral hazard in teams*. A common theme in this field is the focus on free-rider problems due to shared rewards but costly private effort. My paper adds to this literature as it analyzes a dynamic moral hazard problem, in which players have very restricted information about the actions of others, which leads to free-riding and a delay of effort.

This model is also related to the literature on *strategic experimentation* as it models the behavior of players who optimize their decisions while gathering information at the same time. In these games, players have to divide time between a "safe" and a "risky" action (as in the arms of a two-armed bandit) with unknown but common payoffs. Bolton and Harris (1999) analyzes a two-armed bandit problem with many players in which the arms yield payoffs, which behave like a Brownian Motion, with different drifts for the safe and the risky arm. They characterize the unique symmetric Markov Perfect equilibrium and can identify free-rider and encouragement effects. In Keller, Rady, and Cripps's (2005) model of strategic experimentation, the risky arm yields a lump-sum with a certain intensity if the risky arm is good and nothing if the risky arm is bad, new information arrives as a Poisson process, as in most of the recent literature on bandit problems.[4]

[4]A notable exception of this is Boyarchenko (2017), in which Erlang bandits are used.

My work is closely related to Bonatti and Hörner (2011). They analyze a bandit model, similar to Keller, Rady, and Cripps (2005), in which efforts are private information and only outcomes are observable. After success, the game ends, and payoffs are realized.
The model presented in this paper is a very particular model of strategic experimentation: Not only is the information a player gathers about the actions of the other players very restricted, but furthermore, players' payoffs are perfectly correlated. However, strategic experimentation models usually assume the news arrival to be a Poisson process, whereas my model has hardly any restriction on this news arrival process.

Bonatti and Hörner (2011) exemplifies another strand of literature which this paper is related to: *dynamic contribution games.* Already suggested by Schelling (1960), these models analyze the dynamic contributions to public goods. However, unlike in this paper, most papers in this literature either have directly observable contributions (e.g., Admati and Perry (1991) and Lockwood and Thomas (2002)). Others have an information structure that allows deviations from an equilibrium path to be either directly or indirectly observed (e.g., Marx and Matthews (2000)) and thus allow for trigger strategies which are (close to) welfare-maximizing. In the closely related Georgiadis (2014), there is uncertainty about how effort affects the provision of the public good. He assumes that effort affects the drift of a standard Brownian motion towards a (commonly known) threshold and can identify free-riding and encouragement effects and show that the optimal contract only compensates for success. Although in my paper, the uncertainty is about the threshold and not about the effect of effort, the two models are closely related when hazard rates are increasing (see, for example, Example 3). Even closer to this work is Georgiadis (2017), in which he uses a similar model to analyze the effect of monitoring on public good provision. While being very similar in the case without monitoring, his paper explores how even infrequent monitoring can lead to first-best behavior. In contrast, we do not allow monitoring and explore the effect of different types of projects. This paper introduces uncertainty about the effort needed to provide the public good. Therefore, players also have to incorporate information gathering into their decision process. Furthermore, we show that, due to the presence of a deadline, delaying effort is optimal.

There is a vast literature on *procrastination in economics and psychology.* However, these works usually attribute procrastination or delayed effort to self-control problems (O'Donoghue and Rabin (2001)) or time-inconsistencies like hyperbolic discounting (Laibson (1997)). Another explanation for procrastination is given by Akerlof (1991): According to him, procrastination is a consequence of "repeated errors of judgment due to unwarranted salience of some costs and benefits relative to others“ (Akerlof, 1991, p. 3). A notable exception is Weinschenk (2016), in which players also have an incentive to delay their effort, which can be avoided by committing to discriminatory contracts.

The literature on procrastination in psychology is even more prominent than in economics but, like the economic literature, it almost exclusively focuses on some form of cognitive biased decisions (e.g., Wolters (2003) or Klingsieck (2015)).

This paper adds to this literature as it models not only the decision processes of a single person but also delayed effort in teams, i.e., in a game-theoretic model. Furthermore, it explains observed procrastination as rational and even welfare-maximizing behavior and gives rise to entirely different measures that should (or should not) be taken.

Bergemann and Hege (2005) uses a very similar information structure to the one presented in this paper but analyzes a problem in discrete time with linear costs and a memoryless investment. Second, Khan and Stinchcombe (2015) analyze decision problems in which changes can occur at random times and require a costly reaction. They have identified situations in which a delayed reaction is optimal, depending on the hazard rate of the underlying changing probability distributions. This model's relationship to the latter paper is mostly in the use of the hazard rate as a description of the projects players are working on.

To summarize, this paper contributes to the literature in two different ways: On the one hand, it provides a tractable model to analyze a very general class of dynamic contribution games in continuous time with many players and incomplete information about effort contribution. Due to the information structure, we also do not have an equilibrium selection problem. On the other hand, the model can explain the effects of very different types of projects: Projects in which the success probability decreases in effort already spent,[5] e.g., through learning about the quality of the project (which is very common in bandit models), investment projects similar to Georgiadis (2014, 2017) where the past effort increases the chance of success now and even projects in which past effort increases the chance of success on some intervals and decreases on others.

In addition to this, we were able to identify a strategic encouragement effect which can be beneficial or harmful to the effort put towards the project's goal, depending on the type of uncertainties the players are facing. A comparison to the different encouragement effects in the literature is made in Section 4.4.

[5]Covered by decreasing hazard rates of the breakthrough effort distribution.

3. The Model

Consider n risk-neutral players working together on a project in continuous time $t \in [0, T]$, discounting all future payoffs and costs at rate $r > 0$ and with a finite deadline T. We assume that players can only observe their own past effort and whether the project was successful, i.e., they can not observe the other players' effort or the overall effort spent so far. When successful, the players get a lump sum payment, normalized to 1, and the game ends.

A strategy a_i of a player i is a measurable function $a_i : [0, T] \to \mathbb{R}_+$, i.e., how much effort a player spends at every point in time, given that the project was not successful at this time. Effort is costly, and players are assumed to have quadratic instantaneous costs of effort $ca_i(t)^2$, with $c > 0$ at every point in time t.

If the project was not successful before the deadline, T the game ends, and the project can never be completed.

The utility function of player i is, given a breakthrough at time $\bar{t} < T$, therefore given by

$$\tilde{V}_i(a_i, \bar{t}) = re^{-r\bar{t}} - r \int_0^{\bar{t}} e^{-rt} ca_i(t)^2 \, \mathrm{d}t$$

or, if there is no breakthrough before T, by

$$\tilde{V}_i(a_i) = -r \int_0^{T} e^{-rt} ca_i(t)^2 \, \mathrm{d}t.$$

We can see two parts of the utility function here: the first part is the lump sum payment, which occurs only once at time $\bar{t}$ and is therefore discounted by $re^{-r\bar{t}}$. The second part is the cost $ca_i(t)^2$ which occurs at every point in time (and depends on the effort $a_i(t)$) up until $\bar{t}$. The project is successful at time $\bar{t}$ if the players have accumulated enough effort, i.e., more or equal than the threshold $\bar{x}$:

Definition 1 (Effort Threshold)**.** The project is successful at $\bar{t}$ if the players have exerted enough effort to reach the threshold $\bar{x}$, i.e.

$$\bar{t} = \inf \left\{ t' \in [0, T] \,\middle|\, \overline{x} \leq \int_0^{t'} \sum_{i=1}^{n} a_i(t) \, \mathrm{d}t \right\}$$

This definition implies the assumptions of symmetric, additively separable, and linear effects of efforts and non-depreciation of effort.

The threshold $\overline{x}$ is drawn before the game and is unknown to all players. They have a common prior about its probability density function f and hence, about its cumulative distribution function F and hazard rate $h = \frac{f}{1-F}$.

Remark (The hazard rate and projects)**.** *This breakthrough-effort distribution and its hazard rate can be interpreted as the type of a task or project. A project with an increasing hazard rate has a higher chance of being completed the more work already has been put into it (e.g., with learning-by-doing) and vice versa for a decreasing hazard rate. Prior effort does not influence a project with a constant hazard rate.*
Due to the flexibility of the breakthrough-effort distribution, it can incorporate other effects as well; for example, certain types of stochastic project progress or non-deterministic effects of effort. However, throughout this paper, we will stay with the interpretation of a fixed effort threshold and the simple deterministic effort-project progress relation. For more information on the breakthrough-effort distribution and its interpretation as projects, see Appendix C).

Due to Definition 1 we can define $x(t) = \int_0^t \sum_i a_i(s)\,\mathrm{d}s$ as the overall effort already spent up until time t and we can set the initial effort $x(0)$ to 0. Furthermore, $a_{-i}(t) = \sum_{j\neq i} a_j(t)$ is defined as the effort of all players except i at a certain time t.

Therefore, we now know the time $\bar{t}$ at which the project is completed, given an effort profile $\{a_i, a_{-i}\}$:

$$\bar{t} = \inf\left\{ t \geq 0 : \int_0^t \sum_{i=1}^{n} a_i(t)\,\mathrm{d}t \geq \bar{x} \right\}$$

To derive the expected utility, as stated in Equation (1), one has to take the expectations of $\tilde{V}$ with respect to $\bar{t}$. For a detailed derivation please refer to Appendix A.1.

$$V_i\left(a_i(t), a_{-i}(t), x(t)\right) = r\int_0^T e^{-rt}\left(1 - F(x(t))\right)\left[\frac{f(x(t))\left(\sum_j a_j(t)\right)}{1 - F(x(t))} - ca_i(t)^2\right]\mathrm{d}t \qquad (1)$$

The expected utility has an intuitive interpretation: The factor in front of the squared brackets $(1 - F(x(t)))$ gives the probability that we had no success before time t or, in other words, that we reach time t. The two terms in the squared brackets give us the updated belief of the players about having success at time t, given efforts a_j of every player, minus the costs they have to bear.

An increase of instantaneous effort has different effects on the utility of the players: An increase in instantaneous effort increases the chance to win right now (via $\sum_j a_j(t)$) but also increases the instantaneous costs $ca_i(t)^2$. Furthermore, it increases x and thus gives us a chance to end the game right now and neither being able to win nor having to pay any costs in the future (via the $(1 - F(x(t))$ in front of the squared brackets). Finally, spending effort right now changes the chance of winning in the future (conditional on reaching the future) by changing the effect of effort in the future $\frac{f(x(t))}{1-F(x(t))}$. Unlike the effects described before, this effect can go in different directions: effort right now might improve or worsen the effectiveness of effort in the future.

4. Results

4.1. Non-Cooperative Solution

The best response of player i to the strategies of the other players $a_{-i}(t)$ can be stated as the following optimal control problem (omitting the time index t from $x(t)$ and $a_i(t)$):

$$\max_{a_i} V_i = r \int_0^T e^{-rt}(1 - F(x)) \left(\frac{f(x)(a_i + a_{-i})}{1 - F(x)} - ca_i^2 \right) \mathrm{d}t \tag{2}$$

with boundary condition $x_0 = 0$ for the cumulative effort at time 0.

The following assumption restricts our attention to continuous distributions for which there is neither a certain success nor a certain failure.

Assumption 1. The hazard rate of the breakthrough-effort distribution $h(x) := \frac{f(x)}{1-F(x)} > 0$ is continuously differentiable in x and bounded above for every finite x.

Assumption 1 includes a few necessary restrictions on the distribution function to ensure that the optimal control problem, which we have to solve to find the best responses, behaves nicely. The three main parts are $h(x) > 0$, which makes sure that even small amounts of work have a positive probability of yielding a success, continuously differentiability and boundedness of $h(x)$ and therefore $f(x)$.

Note that the assumption explicitly allows for probability mass points at ∞, i.e., that a project might never be successful, independently of the amount of work that the players put into it.

Using this assumption, we can find the unique symmetric Nash equilibrium:

Theorem 1 (Equilibrium Effort)**.** Given Assumption 1, there exists a unique symmetric Nash equilibrium in pure strategies in which, on the equilibrium path, the individual effort of a player a evolves according to

$$\dot{a} = \frac{2n-1}{2} h(x) a^2 + ra - \frac{r}{2c} h(x) \tag{3}$$

and reaches $a_T = \frac{1}{2c} h(x_T)$ at the deadline T.

To find this equilibrium effort path, we use the Pontryagin maximum principle to solve the optimal control problem given by Equation (2) and then use symmetry to find a candidate for the equilibrium effort. Then we show that the path found by the Pontryagin maximum principle always exists and is unique.

To show sufficiency of the solution, we, unfortunately, can not use the "standard" sufficiency conditions (i.e., global concavity or Arrow/Mangasarian sufficiency conditions) due to the general breakthrough-effort distribution $f(x)$.

However, we can show that a global maximizer always exists through a Tonelli-type existence theorem. Furthermore, using a result from optimal control theory, we can show that every maximizer has to be a Pontryagin extremal (i.e., must be one of the candidates we have found above).[6]

Now, we know that a global maximizer exists, that every maximizer has to be a Pontryagin extremal, and that there is only one Pontryagin extremal. Therefore, we know that our Pontryagin extremal has to be the unique global maximizer of our problem and is, therefore, the unique symmetric Nash equilibrium.

[6]To people familiar with optimal control problems, this might be surprising. While we are using a result from optimal control theory to show this, the main reason for this is that we have excluded boundary arcs due to the assumptions. An intuitive interpretation for this is that due to the boundedness and continuity of $h(x)$, the value of additional effort is bounded, whereas the costs are not.

Remark. *Every Nash equilibrium is due to the game's information structure, sequentially rational, and a Perfect Bayesian equilibrium (for some beliefs, e.g., the correct ones). At every point in time, every player can only choose an action at exactly one information set.*

Remark (Asymmetric equilibria)**.** *While there might be many asymmetric equilibria, we are only considering symmetric equilibria in this paper. Given the symmetric setting and the information structure, restricting our attention to symmetric equilibria seems natural. Furthermore, every welfare maximizing solution to the problem is, due to the quadratic costs, symmetric (see Section 4.2).*

In the following, we are going to focus our attention on equilibrium behavior. However, let me briefly discuss an idea of the **off-equilibrium behavior** that arises if one player deviates from the equilibrium path. If she exerts less effort at time t, her continuation strategy after t depends on the hazard rate of the underlying breakthrough effort distribution:

- Given a **constant hazard rate**, nothing changes for her. In this special case, the behavior is independent of the past; hence she will immediately revert to the equilibrium effort.
- Given an **increasing hazard rate**, her belief about the probability of success is now lower than that of her collaborators. Therefore, she will also exert less effort in the future. Given a steep enough slope of the hazard rate, this leads to divergence of her belief (and therefore effort).
- Given a **decreasing hazard rate**, her belief about the probability of success is now higher than that of her collaborators. This leads to a higher effort until her belief coincides again with the other players' beliefs, as soon as she has made up the effort she previously failed to exert. So, given enough time, in this case, the player will revert to the symmetric equilibrium.

Using Theorem 1, we can show that effort is increasing over time for certain types of projects and if there is no discounting.

Proposition 1 (Increasing Effort)**.** The equilibrium effort is increasing everywhere for non-decreasing hazard rates and decreasing hazard rates if the discount rate r is 0.

For the proof, see Appendix A.3.

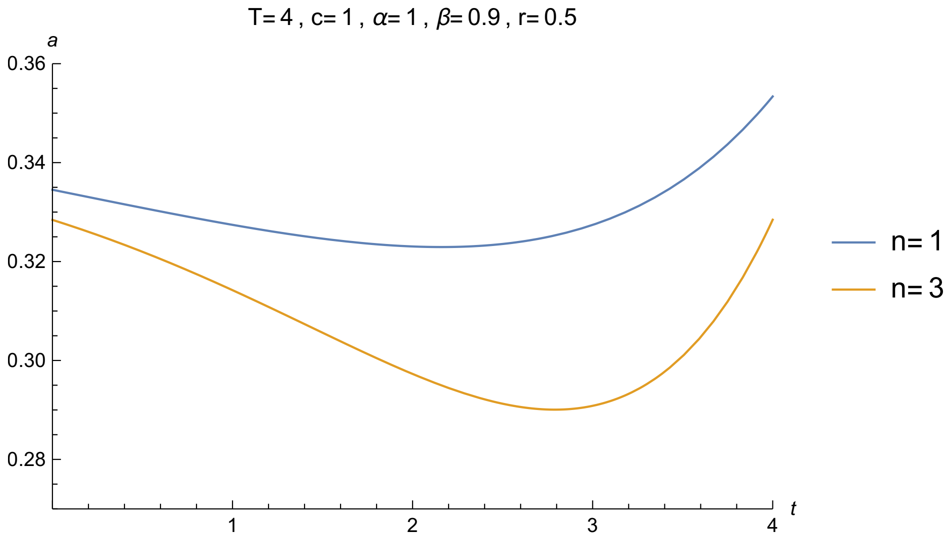

Figure 1: Equilibrium effort: Decreasing hazard rate

However, suppose the project has a decreasing hazard rate. In that case, the effort might also be decreasing (see for example Figure 1).[7]

4.2. Welfare-Maximizing Solution

To solve the problem of the social planner, we have to solve a problem similar to Equation (2). However, now we maximize the combined utility and therefore:

$$\max_{a_i} V_i = r \int_0^T e^{-rt}(1 - F(x)) \left(n \frac{f(x)(\sum_i a_i)}{1 - F(x)} - \sum_i c a_i^2 \right) \mathrm{d}t \tag{4}$$

We can focus on the symmetric problem in which every player exerts $\bar{a}(t)$ without loss of generality, as the following Lemma shows us.

Lemma 1. Every welfare-maximizing effort path has to be symmetric.

The intuition for Lemma 1 is as follows: Due to the assumptions of symmetric and additive-separable effects of efforts (Definition 1) and convex costs, an equal distribution of the efforts exerted at every point in time results in the same probability of success but a lower sum of costs. For the proof, see Appendix A.4.

[7] Here, $f(x)$ is an incomplete exponential distribution, i.e., an exponential distribution with rate parameter α and a probability mass point at ∞ of $1 - \beta$.

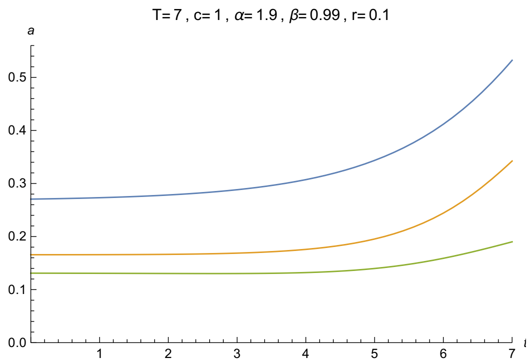

Figure 2: Nash equilibrium effort

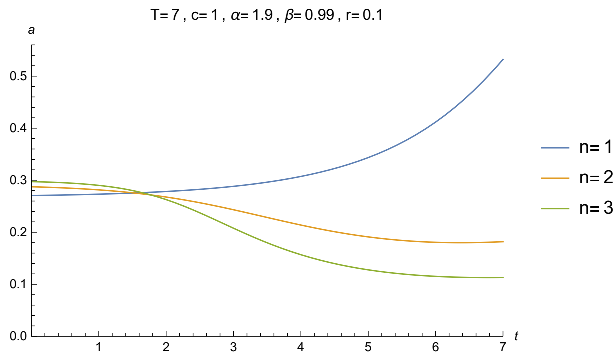

Figure 3: Welfare-maximizing effort

Therefore we get the social planners optimization problem

$$\max_{\bar{a}} V_i = r \int_0^T e^{-rt}(1 - F(x)) \left(\frac{f(x)(n^2 \bar{a})}{1 - F(x)} - nc\bar{a} \right) \mathrm{d}t \tag{5}$$

And its solution which is derived in a similar fashion to Theorem 1 in Appendix A.5.

Proposition 2 (Welfare-Maximizing Effort)**.** In the social optimum, every player exerts an effort which evolves according to

$$\dot{a} = \frac{1}{2} h(x) a^2 + ra - \frac{nr}{2c} h(x)$$

and reaches $a_T = \frac{n}{2c} h(x_T)$ at time T.

One might suspect that the welfare-maximizing solution is to always exert more effort than in the equilibrium. While this can be observed with an increasing or a constant hazard rate, it is not true for decreasing hazard rates, as shown in Figures 2 and 3 (which uses the incomplete exponential distribution with success rate α and failure rate $1 - \beta$). Here we can see that the welfare-maximizing effort starts off being higher (and the cumulative effort is always higher). Due to the decreasing belief in the project's success, effort decreases much faster than in the equilibrium.

Proposition 3 (Comparison Between Equilibrium and Welfare-Maximizing Effort Levels)**.**

(a) The cumulative welfare maximizing effort x^* is at least as high as the cumulative equilibrium effort x at every point in time t.

(b) The welfare-maximizing effort a^* always starts higher than the equilibrium effort a but might be overtaken by it later on.

(c) The welfare-maximizing effort a^* for a non-decreasing hazard rate is, near the deadline, always higher than the equilibrium effort a.

For the proof see Appendix A.7.

4.3. Last-minute Rush

In reality, we often observe the so-called last-minute rush. While this is often contributed to irrational behavior (e.g., procrastination), we also observe this model's effect with rational players.

Definition 2 (Last-minute rush)**.** Player i exhibits a last-minute rush if and only if $\exists \delta > 0$ s.t. a_i is increasing on $[T - \delta, T]$.

Given Theorem 1, we can show that a last-minute rush can be observed for every variation of the model:

Theorem 2 (Last-minute rush)**.** For every possible breakthrough-effort distribution that fulfills Assumption 1, players delay their effort, as defined in Definition 2, on the symmetric equilibrium path.

For the complete proof see Appendix A.6.

Surprisingly, the last-minute rush is present for any breakthrough-effort distribution, even when the hazard rate decreases sharply near the deadline.

We know that we have a last-minute rush in the equilibrium, but what about the social optimum? The following proposition shows that we can expect a rational social planner to delay effort.

Proposition 4 (Last-minute rush of the social planner)**.** For every possible breakthrough-effort distribution that fulfills Assumption 1, the social planner delays the effort, as defined in Definition 2.

The proof of Proposition 4 is analogous to the proof of Theorem 2.

Corollary (Comparison of the cooperative and non-cooperative last-minute rush)**.** For non-decreasing hazard rates, we can compare the last-minute rush between the cooperative and non-cooperative last-minute rush:

1. $\exists \delta : a* > a \quad \forall t \in [T - \delta, T]$
2. $\exists \delta : \dot{a}^*(t) > \dot{a}(t) \quad \forall t \in [T - \delta, T]$

We can see that the last-minute rush is not only something that also occurs in the welfare-maximizing solution. For non-decreasing hazard rates, it is even stronger, at least in absolute terms.

On the other hand, the increase towards the end might even be smaller for decreasing hazard rates. This is not surprising, as a decreasing hazard rate means that the more effort was put into the project already, the lower the chance of future success, given the same future effort. With the result above, it is easy to imagine a case in which the chance to succeed in the cooperative solution is very close to zero and much higher in the non-cooperative solution.

4.4. Free-Riding, the Encouragement Effect and Delay of Effort

In this part of the paper, we will see the three effects driving the behavior in this model: free-riding, the encouragement effect, and delay of effort.

Free-riding, i.e., the effort level is inefficiently low due to externality problems, has two causes in this model: One is that my efforts have a positive externality on other players' payoffs (which can be seen in Equation (1)) and that efforts are strategic substitutes, i.e., as I know that the other players will also work, I will work less (for a proof that efforts are strategic substitutes at every point in time see Appendix A.8). This leads to lower than optimal (i.e., welfare-optimizing) efforts, which we have already seen in Proposition 3.

However, my effort does affect not only the instantaneous effort choice of the other players but also the perceived *effectiveness* of effort for every player in the future and therefore their choice of effort indirectly.[8] We call this effect **encouragement effect**, as players might exert more effort for a given belief level as their effort might encourage others to exert more effort

[8] As players can not observe the cumulative effort directly in this game, the effect works via the beliefs of the players: In the equilibrium, I have a correct belief about x, which influences my decision.

in return. While free-riding is a well-known effect, the encouragement effect is, while related to similar effects in the literature, new.[9]

Formally, we define it as the change of the best responses to the past effort, i.e., as $\frac{\partial a_i^*(\cdot)}{\partial x(t)}$. This captures the idea of "One player's experimentation leads to the other player experimenting more later which, in turn, is valuable to the first player."

As we will see, it is able to link strategic effects to different kinds of projects:

Theorem 3 (The encouragement effect)**.**

- For an increasing hazard rate, we observe a positive encouragement effect.
- For a constant hazard rate, we observe no encouragement effect.
- For a decreasing hazard rate, we observe a negative encouragement (discouragement) effect.

For the proof see Appendix A.9.

The effect of the encouragement effect can go in two directions:
For an increasing hazard rate, the effect is called encouragement effect for a good reason. Every effort a player spends now increases the effectiveness and, therefore, everyone's effort in the future. However, with a decreasing hazard rate, this effect is a negative encouragement or discouragement effect: If a player spends more effort now and we do not succeed, we have a lower belief about the chance of succeeding in the future and will therefore work less. This leads to less effort, especially in the earlier periods.

The encouragement effect in the literature

The definition of the encouragement, as defined here, follows the idea of Bolton and Harris (1999), is, however, defined differently. The reason for this is that the models are fundamentally different in several aspects. Their definition is that experimentation does continue after the belief threshold under which a single agent would stop experimenting. We can not apply their definition for two reasons: In this model, agents never stop experimenting before the deadline or success. Furthermore, agents are not using Markovian strategies, and therefore experimentation can increase, for example, due to being close to the deadline (see below).
However, the idea of encouragement in the sense of Bolton and Harris (1999) described as "[A]n individual player may be encouraged to experiment more if, by so doing, she can bring

[9]For a comparison of our encouragement effect to different effects in the literature, see the remark further down.

forward the time at which the information generated by the experimentation of others becomes available.“ is covered by the above definition.

Very close to the effect in this paper is the effect described as encouragement effect in Georgiadis (2014). In his paper, the projects described are, in the words of this paper, projects with an increasing hazard rate, which explains why he always observes an encouragement effect and no discouragement effect.

A similar but unrelated effect (called encouragement) can be found in Keller, Rady, and Cripps (2005), which depends on allowing asymmetric strategies that allow for more efficient equilibria. As this paper only considers symmetric strategies, we can not observe this effect here.

Another way encouragement occurs if players can influence other players' beliefs over the project: For example, in Dong (2018), asymmetric information can lead to more effort exerted by the better-informed player to increase the beliefs of the less-informed player. In Cetemen, Hwang, and Kaya (2020), players do not learn about the project's quality via experimentation but through an exogenous feedback function. This feedback function is increasing in the project's quality and the combined past effort of all players, thus linking beliefs and efforts.

However, even without the encouragement effect, the question of when to work is not trivial. On the one hand, the discount rate incentivizes players to work earlier, while the convex costs make spreading the effort over a larger time more efficient. If players assume an increasing hazard rate, we should expect them to work harder closer to the deadline, as the effectiveness of their effort increases. But what does the optimal effort for a decreasing hazard rate look like? From the example depicted in Figure 1, we already get a good idea of what the typical optimal effort path might look like: At first, we have a decrease in effort. This is due to the encouragement effect: Early on, the effectiveness of effort is high, but due to the decreasing hazard rate, the success rate and the effort level decrease. However, later on, the effort increases again, despite the effectiveness being at its lowest. Therefore, we show there is a third effect at work: a **delay of effort**.

We have already seen from the previous section that, close to the deadline, delay of effort dominates every other effect near the deadline, not only in the equilibrium but also in the welfare-maximizing solution. While this shows that delay of effort is not a strategic effect, it still has severe implications on working in teams: Delaying effort is not only expected but also efficient and beneficial to the team. Therefore, team members and leaders should take that into account when evaluating teamwork.

What might be the reason for this behavior? This effect is independent of the discount rate, as it also occurs with patient players (i.e., if $r = 0$, see Appendix E) and disappears without a deadline (see, for example, Appendix D). Furthermore, we do not observe it in Bonatti and Hörner (2011), a very similar model with linear instead of quadratic costs.[10]

Therefore, it is a consequence of convex costs and a deadline.

The intuition for this result is that players try to distribute effort as evenly as possible due to the convex costs. But every time they invest effort without being successful, they have to update their belief over the effort threshold upwards. Thus, they are smoothing their effort again, now with a higher target effort level and less time to distribute, as the deadline is closer.

But what is the intuition behind the result that delay of effort effect stronger than every other effect when close to the deadline?
That an increasing hazard rate pushes effort even more towards the end is not surprising: As they have invested effort before, they are getting a higher chance of success for the same amount of effort than before. Thus, they are willing to spend more effort on it. Furthermore, the encouragement effect also goes in the same direction, the more effort has been spent, the more effort I am willing to spent.

However, for a decreasing hazard rate, these two effects are going in the opposite direction: The encouragement effect is a discouragement effect, so effort should be lower. However, the encouragement effect is forward-looking (i.e., my effort now affects others' effort later) and is getting weaker the closer one is to the deadline. At the deadline T, it even vanishes entirely. But the effect of decreasing hazard rates itself neither forward-looking nor a strategic effect: The more work has been put into a project, the lower the effectiveness of effort (i.e., players have a lower chance of success for the same amount of effort). However, this effectiveness is bounded away from zero by Assumption 1. Now, in the limit $t \to T$, the costs approach 0, whereas the reward of success is constant. Thus, when being close enough to the deadline, the effect of the decreasing hazard rate has to be overcome by the effect of the costs.

[10]In their model, the welfare-maximizing effort is as follows: As the chance of success decreases in the invested effort, players invest the maximal amount of effort until the marginal benefits of effort are lower than the marginal costs. After that point is reached, no effort is invested anymore.

4.5. Certain Objectives

Let's have a brief look at the model's analysis without uncertain objectives, i.e., if the threshold $\bar{x}$ is common knowledge. This case is very similar to the model of Georgiadis (2017) without monitoring. In this part of the paper, we will normalize $c = \frac{1}{2}$ as in the paper mentioned above to make the comparison easier. The major difference is that rewards are paid out as soon as the project is completed in our model. Whereas in his model, rewards are paid out after a monitoring event (i.e., in this case, at the deadline T). Therefore, there is no incentive to complete the project early in his model, whereas, in this paper, there is a strong incentive to complete the project as soon as possible.

Unfortunately, as Assumption 1 is not fulfilled anymore, as we have a probability mass point of 1 at the known threshold and $h(x) = 0$; otherwise, most of the results obtained in this work do not hold anymore. We know that multiple equilibria exist, for example, equilibria in which no one works.

However, restricting ourselves to symmetric project-completing equilibria (i.e., $x_t = \bar{x}$ for some $t \leq T$) and using Georgiadis (2017, Proposition 3), we can find a symmetric equilibrium:

Proposition 5 (Known Effort Threshold)**.**
If the effort threshold is known, the project is profitable (i.e., the expected value for each player is positive, or $r\bar{x}^2 < 2n^2$), either

- constant effort is exerted between $t = 0$ and $t^* = T$ (for $r = 0$) or
- the effort is increasing between $t = 0$ and some $t^* \leq T$ (for $r > 0$) and 0 otherwise:

$$a_i = \frac{r\bar{x}}{n} \frac{e^{rt}}{e^{rt^*} - 1} \quad \forall t \in [0, t^*]$$
$$t^* = \min \left\{ \frac{1}{r} \log \left(n \left(\frac{\sqrt{2}\bar{x}\sqrt{r}}{2n^2 - \bar{x}^2 r} + \frac{2n}{2n^2 - \bar{x}^2 r} \right) \right), T \right\}$$

The proof splits the problem into two parts: the optimal effort allocation, which has already been solved by Georgiadis (2017, Section 5.1) and one of finding the optimal t^*.

A brief discussion of the proof can be found in Appendix B.

Again, the equilibrium depicted is only one of many possible equilibria, but it shows two important things: In our model, unlike in Georgiadis (2017), discounting, even under a known effort threshold, might lead to early completion of the project. Furthermore, with a known effort threshold, we only observe an increasing effort towards the end due to discounting and usually not for very profitable projects or late deadlines, as these tend to be completed before the deadline.

Therefore, while we can see an increasing effort in many examples without uncertain objectives, it is due to discounting and not the delay of effort described above.

5. Conclusion

In this paper, we have analyzed a team working together on a project where the individual team members are unable to observe each others' efforts and have only a rough idea about the amount of effort that will be needed to complete the project.

In this model, we can observe an encouragement effect and a delay of effort. The delay of effort analyzed here is not a result of inefficient behavior but a necessary consequence of the deadline and convex costs, given the information structure. With the encouragement effect, we are now able to link different types of projects to strategic behavior.

Within this model, we can also analyze the influence of the model's parameters on the behavior of the players and the social planner. However, this paper's focus is to give a model of uncertain objectives in a setting with strategic experimentation and the resulting encouragement effect and the delay of effort. Furthermore, we discuss the special cases of patient players (i.e., when players have a discount rate of $r = 0$, in Appendix E), the effect of team size (Appendix G) and of certain objectives (i.e., if the breakthrough-effort threshold $\bar{x}$ is known, in Appendix B) and give an example of a case in which uncertain objectives improve the overall welfare (Appendix D.1).

This paper opens up questions for future research. For example, the question if the result of Bonatti and Hörner (2011) that deadlines might improve welfare carries over to the case of convex costs is still unanswered.[11] Another interesting direction is the analysis of the optimal compensation scheme for different types of projects.

The effects observed in this model are not only able to explain frequently observed behavior in the workplace like procrastination, they also advise on how to organize teamwork when working on projects with a deadline: First, delaying effort isn't necessarily bad for the team and thus does not has to be sanctioned by the team leader. Furthermore, the encouragement

[11] Simulations (see Appendix F) suggest that improved welfare due to deadlines is a result of linear costs and might not carry over to quadratic costs.

effect shows us that working in teams is less efficient when the project has (or is perceived to have) a decreasing hazard rate. To avoid this, one should avoid giving projects with an imminent change of failure to larger groups.

Appendix

A. Proofs

A.1. Derivation of the Expected Utility

We know that the breakthrough effort is drawn from a distribution with the probability distribution function f and the cumulative distribution function F and that $x(t)$ is by Definition 1

$$x(t) = \int_0^t a_i(s) + a_{-i}(s)\,\mathrm{d}s$$

with $a_{-i}(s) = \sum_{j \neq i} a_j(s)$. The breakthrough time $\bar{t}$ is the first time at which enough effort (i.e., the breakthrough effort) is accumulated:

$$\bar{t} = \inf\{t \geq 0 | x(t) \geq \bar{x}\}.$$

The utility is then, given some fixed breakthrough time $\bar{t}$

$$\tilde{V}_i(a_i, \bar{t}) = re^{-r\bar{t}} - r \int_0^{\bar{t}} e^{-rt} c a_i(t)^2\,\mathrm{d}t.$$

Therefore, we know that the payoff part ($re^{-r\bar{t}}$) of the expected utility is equal to the distribution of $\bar{t}$: $\bar{f}(t)$. Let us now rewrite the PDF and CDF of $\bar{t}$ as a distribution over x:

$$\bar{F}(t) = P[t \geq \bar{t}] = P[x(t) \geq \bar{x}] = F(x(t)) \Rightarrow \bar{f}(t) = f(x(t))\,(a_i(t) + a_{-i}(t))\,.$$

Therefore, the expected payoff is

$$\mathbb{E}_{\bar{t}}\left[re^{-r\bar{t}}\right] = r \int_0^T e^{-rt} f(x(t))\,(a_i(t) + a_{-i}(t))\,\mathrm{d}t. \tag{6}$$

For the expected costs $r \int_0^{\bar{t}} e^{rt} c a_i(t)^2\,\mathrm{d}t$, we have to distinguish between two cases: One in which the project is successful (i.e., $\bar{t} < T$) and we pay until $\bar{t}$ and one in which it is unsuccessful ($\bar{t} \geq T$) and we only pay until T. As we know that $1 - \bar{F}(\infty) = P[\bar{t} = \infty] = P[\bar{x} > x(\infty)] = 1 - F(x(\infty))$ we get:

$$\begin{aligned}
&\int_0^\infty \int_0^{\min\{t,T\}} e^{-rs} ca(s)^2 \, \mathrm{d}s \, \mathrm{d}F(x(t)) + (1 - F(x(\infty)) \int_0^T e^{-rs} ca(s)^2 \, \mathrm{d}s \\
&= \int_0^\infty \int_0^\infty \mathbb{1}_{s<t} \mathbb{1}_{s<T} e^{-rs} ca(s)^2 \, \mathrm{d}s \, \mathrm{d}F(x(t)) + (1 - F(x(\infty)) \int_0^T e^{-rs} ca(s)^2 \, \mathrm{d}s \\
&\overset{\text{Fubini's Theorem}}{=} \int_0^\infty \int_0^\infty \mathbb{1}_{s<t} \mathbb{1}_{s<T} e^{-rs} ca(s)^2 \, \mathrm{d}F(x(t)) \, \mathrm{d}s + (1 - F(x(\infty)) \int_0^T e^{-rs} ca(s)^2 \, \mathrm{d}s \\
&= \int_0^\infty \mathbb{1}_{s<T} e^{-rs} ca(s)^2 \int_0^\infty \mathbb{1}_{s<t} \, \mathrm{d}F(x(t)) \, \mathrm{d}s + (1 - F(x(\infty)) \int_0^T e^{-rs} ca(s)^2 \, \mathrm{d}s \\
&= \int_0^\infty \mathbb{1}_{s<T} e^{-rs} ca(s)^2 (1 - F(x(s)) \, \mathrm{d}s \\
&= \int_0^T e^{-rs} ca(s)^2 (1 - F(x(s)) \, \mathrm{d}s. \qquad (7)
\end{aligned}$$

If we add the expected payoff (Equation (6)) and the expected costs (Equation (7)), we get the expected utility, as stated in Equation (1):

$$V_i = r \int_0^T e^{-rt} (1 - F(x(t))) \left(\frac{f(x(t))(\sum_j a_j(t))}{1 - F(x(t))} - ca_i(t)^2 \right) \mathrm{d}t.$$

A.2. Theorem 1 (Optimal Effort)

Candidate Solution

Finding the best response a_i of some player i to the strategies of the other players a_{-i} in the problem stated in Equation (2) is a discounted optimal control problem of the following form

$$
\begin{aligned}
H(x, \lambda, t) &= f(x)(a_i + \sum_{j \neq i} a_j) - c a_i^2 (1 - F(x)) + \lambda(a_i + a_{-i}) \\
\dot{x} &= a_i + \sum_{j \neq i} a_j \\
x(0) &= 0 \\
\lambda(T) &= 0 \\
a_i, x &\in \mathbb{R}_+ \quad \forall i
\end{aligned}
\tag{8}
$$

Using the Pontryagin maximum principle (Pontryagin, Boltyanskii, Gamkrelidze, and Mischenko (1962)) in the version of Kamien and Schwartz (2012), we know that the necessary conditions for a maximum are

$$\frac{\partial H}{\partial a} = 0 \tag{9}$$

$$\frac{\partial H}{\partial x} = r\lambda - \dot{\lambda} \tag{10}$$

$$\frac{\partial H}{\partial \lambda} = \dot{x} \tag{11}$$

$$\lambda(T)x(T) = 0 \quad \Rightarrow \quad \lambda(T) = 0 \tag{12}$$

with the Hamiltonian (Equation (9)), the equation of motion for the state variable (Equation (10)), the equation of motion for the costate variable (Equation (11)) and the transversality condition (Equation (12)) for $x(T)$ being free. In addition we can see that the optimal control does not depend on the a_j's of the other players but only on the sum $a_{-i} := \sum_{j \neq i} a_j$. Therefore we get

$$
\begin{aligned}
a_i &= \frac{f(x) + \lambda}{2c\,(1 - F(x))} \\
\dot{\lambda} &= r\lambda - f'(x)(a_i + a_{-i}) - c a_i^2 f(x) \\
\dot{x} &= a_i + a_{-i} \\
x(0) &= 0, \quad \lambda(T) = 0
\end{aligned}
$$

From here on, we only consider symmetric equilibria, therefore we can replace a_{-i} by $(n-1)a_i$.

Hence, necessary conditions for a best response are:

$$\begin{aligned}
a_i &= \frac{f(x)+\lambda}{2c\left(1-F(x)\right)} \\
\dot{\lambda} &= r\lambda - f'(x)na_i - ca_i^2 f(x) \\
\dot{x} &= na_i \\
x(0) &= 0, \quad \lambda(T) = 0.
\end{aligned}$$

Using a_i we get

$$\begin{aligned}
\dot{\lambda} &= r\lambda - f'(x)n\left(\frac{f(x)+\lambda}{2c\left(1-F(x)\right)}\right) - c\left(\frac{f(x)+\lambda}{2c\left(1-F(x)\right)}\right)^2 f(x) \\
\dot{x} &= n\left(\frac{f(x)+\lambda}{2c\left(1-F(x)\right)}\right) \\
x(0) &= 0, \quad \lambda(T) = 0.
\end{aligned}$$

So, the equation of motion for the costate and its time derivative are

$$\begin{aligned}
\lambda &= \frac{2c}{n}\left(1-F(x)\right)\dot{x} - f(x) \\
\dot{\lambda} &= \frac{2c}{n}\left(1-F(x)\right)\ddot{x} - \frac{2c}{n}f(x)\dot{x}^2 - f'(x)\dot{x}
\end{aligned}$$

Using this, the necessary conditions simplify to the following boundary value problem:

$$\begin{aligned}
\left(1-F(x)\right)\ddot{x} &= -\frac{nr}{2c}f(x) - \frac{1}{2n}f(x)\dot{x}^2 + f(x)\dot{x}^2 + r\left(1-F(x)\right)\dot{x} \\
x(0) &= 0 \\
\lambda(T) &= \frac{2c}{n}\left(1-F(x_T)\right)\dot{x}_T - f(x_T) = 0
\end{aligned}$$

Introducing the hazard rate $h(x) := \frac{f(x)}{1-F(x)}$, we have necessary conditions for Equation (2)

$$\begin{aligned}
\ddot{x} &= -\frac{rn}{2c}h(x) + \frac{2n-1}{2n}h(x)\dot{x}^2 + r\dot{x} \\
x(0) &= 0 \\
\dot{x}_T &= \frac{n}{2c}h(x_T)
\end{aligned} \tag{13}$$

Or, in terms of the individual effort, i.e., using that $\dot{x} = na$ and $\ddot{x} = n\dot{a}$:

$$\begin{aligned}\dot{a} &= -\frac{r}{2c}h(x) + \frac{2n-1}{2}h(x)a^2 + ra \\ a_T &= \frac{1}{2c}h(x_T)\end{aligned} \tag{14}$$

which is a non-linear boundary value problem of the second order.

However, a solution to this problem might not exist, and even if it exists, it might not be unique. The following lemma shows that it does exist uniquely.

Lemma 2 (Existence and Uniqueness of the Solution to the Initial Value Problem)**.** A solution to the initial value problem from Equation (13) (and therefore also for Equation (14)) exists and is unique.

Proof. As $a_i : [0, T] \rightarrow \mathbb{R}_+$ is continuous and maps from a compact space to a metric space, we know that it is bounded. Therefore, $\frac{r}{2c}h(x) + \frac{2n-1}{2n}h(x)a^2 - ra$ is Lipschitz-continuous in (a, x) as a is bounded (and therefore also a^2), $x(t) = \int_0^t na(s)\,\mathrm{d}s$ and $x_0 = 0$ and $h(x)$ is bounded due to Assumption 1. Thus (by Picard-Lindelöf), we know that a unique solution to the initial value problem for a_i exists.

As $x(t) = \int_0^t na(s)\,\mathrm{d}s$ and $x_0 = 0$ and a exists and is unique, $x(t)$ also exists uniquely. □

This does not mean that the Nash equilibrium exists. The candidate solution might not be a global maximizer to the problem (i.e., the conditions in Equation (14) might not be sufficient). Furthermore, even if Equation (14) represents a Nash equilibrium, there might be other Nash equilibria. There could be, for example, boundary arcs or other solutions that might not be found by the Pontryagin maximum principle.

Therefore, we have to show sufficiency and uniqueness of the solution.

Sufficiency and uniqueness of the solution

To do that, we first show that our optimal control problem has at least one global maximizer via a Tonelli-type existence proof. Then, we show that every maximizer has to be a Pontryagin extremal, i.e., will be found by the Pontryagin maximum principle. Thus we know that the candidate solution is the only maximum as the candidate solution is unique.

To show the existence, we use a Tonelli-type existence theorem. For convenience, we use the version from Torres (2004), Theorem 10, as we are using results from this paper later on.

Lemma 3 (Existence of a Global Maximizer)**.** The maximization problem, as stated in Equation (2), has an absolute maximum in the space of the control.

Proof. Following Torres (2004), we have to show that the problem, to have a global minimum, has to satisfy **Coercivity** and **Convexity**.

Following the notation of the paper, $-L(t,x,a) = -\left(f(x)(a_i + \sum_{j\neq i} a_j) - ca_i^2(1 - F(x))\right)$ is the Lagrangian of our problem from Equation (8) multiplied with -1 to transform the maximization problem into a minimization problem and $\varphi(t,x,a) = a_i + \sum_{j\neq i} a_j$ is the evolution of the state.

Then we know that a global maximizer always exists if the following conditions exist for all (t,x,a):

Coercivity: There exists a function $\theta : \mathbb{R}_+ \longrightarrow \mathbb{R}$, bounded below, such that:

$$\lim_{r->+\infty} \frac{\theta(r)}{r} = +\infty,$$
$$-L(t,x,a) \geq \theta(\varphi(t,x,a)),$$
$$\lim_{a_i->+\infty} \varphi(t,x,a)) = +\infty.$$

As $\varphi(t,x,a)) = a_i + \sum_{j\neq i} a_j$ is linear in a_i, the last condition is fulfilled. Now, we have to show that there exists a function θ that fulfills the other two conditions.

$-L(t,x,a) = -\left(f(x)(a_i + \sum_{j\neq i} a_j) - ca_i^2(1 - F(x))\right) = ca_i^2(1 - F(x)) - f(x)(a_i + \sum_{j\neq i} a_j)$ is, as we know from Assumption 1 that $f(x)$ is bounded above and $1 - F(x)$ below, bounded and we can always find a $\theta()$, s.t. $\theta(a_i) \leq -L(t,x,a)$ and $\lim_{r->+\infty} \frac{\theta(r)}{r} = +\infty$.[12]

Convexity: $-L(t,x,a) = -\left(f(x)(a_i + \sum_{j\neq i} a_j) - ca_i^2(1 - F(x))\right)$ and the evolution of the state $\varphi(t,x,a) = a_i + a_{-i}$ both have to be convex in a_i.[13]

[12] For example $\theta(r) = \alpha r^2 - \beta r - \gamma$ with $\alpha \leq c(1 - F^-), \beta \geq f^+$ and $\gamma > f^+ a_{-i}$ with f^+ being the upper bound of f and F^- the lower bound of F fulfills the conditions.

[13] For existence, we do not need global convexity or convexity in the state.

We have assumed that $c > 0$ and that $f(x) > 0, F(x) < 1$ follows directly from Assumption 1. Thus $L(t, x, a)$ is concave in a_i (and $-L(t, x, a)$ convex in a_i). Furthermore, $\varphi(t, x, a)$ is linear in a_i and therefore also convex.

Thus, as Convexity and Coercivity are given, we know that our optimal control problem has a global maximizer. □

However, this does not ensure that we can find this maximizer using the Pontryagin maximum principle. For example, we might have singular arcs or upper or lower boundary arcs.[14]

We establish that the maximizer we found is, in fact, the global maximizer by using the main result of Torres (2004) (Theorem 13 and Corollary 4):

Lemma 4 (Maximizers are Pontryagin extremals)**.** All maximizers of the problem as stated in Equation (8) are Pontryagin extremals.

Proof. Using Torres (2004) Corollary 4, we know that Coercivity (which we have shown in Lemma 3) and the following conditions imply that all maximizers are found be the Pontryagin maximum principle, if there exist constants $k_1 > 0$ and k_2 such that

$$\left|\frac{\partial L}{\partial t}\right| \le k_1|L| + k_2, \quad \left|\frac{\partial L}{\partial x}\right| \le k_1|L| + k_2,$$
$$\left|\frac{\partial \varphi}{\partial t}\right| \le k_1|\varphi| + k_2, \quad \left|\frac{\partial \varphi_{a_i}}{\partial x}\right| \le k_1|\varphi_{a_i}| + k_2.$$

The first inequality is trivially fulfilled. as L does not depend on t. The second inequality is fullfilled, as $h(x)$ and therefore also $f(x)$ are continuously differentiable (Assumption 1). The third inequality is fulfilled, as φ does not depend on t and, as φ (and thus φ_{a_i}) is independent of x, the last inquality is also fulfilled. □

Thus, we know that every maximizer has to be found by the Pontryagin maximum principle and that at least one global maximizer exists. Furthermore, we know that Equation (14) is the solution found by the Pontryagin maximum principle and that it is the unique solution to the Pontryagin maximum principle. Therefore, the following corollary follows.

[14]We would have, for example, a lower boundary arc if a player would stop working at some point in time.

Corollary (Sufficiency and uniqueness)**.** The solution to Equation (14) gives us the unique global maximizer to the optimal control problem in Equation (8) and, therefore, the unique symmetric Nash equilibrium.

Thus we know that the initial value problem describes the only symmetric Nash equilibrium in pure strategies in Equation (13).

A.3. Proposition 1 (Increasing Effort)

Proof. **Non-decreasing hazard rates**
Assume your effort is decreasing at some point t_1, i.e., $\dot{a}_1 < 0$. From Theorem 2 we know that there is a t_3 close to T, such that $\dot{a}_3 > 0$. Therefore, due to continuity of a there exists a t_2 such that $t_1 \leq t_2 \leq t_3$ with $\dot{a}_2 \geq 0$ and $a_2 \leq a_1$. As we have a non-decreasing hazard rate we know that $h(x_2) \geq h(x_1)$. Thus, we have for $r \neq 0$:

$$
\begin{aligned}
& \dot{a}_1 && < 0 \\
\Leftrightarrow \quad & \frac{2n-1}{2} h(x_1) a_1^2 + r a_1 - \frac{r}{2c} h(x_1) && < 0 \\
\xRightarrow{r \geq 0 \text{ and } a_2 \leq a_1} \quad & \frac{2n-1}{2} h(x_1) a_1^2 + r a_2 - \frac{r}{2c} h(x_1) && < 0 \\
\xRightarrow{a_2 \leq a_1} \quad & \left(\frac{2n-1}{2} a_2^2 - \frac{r}{2c} \right) h(x_1) + r a_2 && < 0 \\
\xRightarrow{h(x_2) \geq h(x_1), h(x_1) \geq 0, r > 0 \text{ and } a_2 > 0} \quad & \left(\frac{2n-1}{2} a_2^2 - \frac{r}{2c} \right) h(x_2) + r a_2 && < 0 \\
\Rightarrow \quad & \frac{2n-1}{2} h(x_2) a_2^2 + r a_2 - \frac{r}{2c} h(x_2) && < 0 \\
\Leftrightarrow \quad & \dot{a}_2 < 0 && \textbf{Contradiction}
\end{aligned}
$$

r=0
For $r = 0$ $\dot{a} > 0$ results trivially from Theorem 1, as $\frac{2n-1}{2} h(x) a^2 \geq 0$ □

A.4. Lemma 1 (Asymmetric Equilibria)

Proof. Assume there is an asymmetric equilibrium that is welfare maximizing. Then $\exists i, t, j :$ $a_i(t) > a_j(t)$. However, it would be possible to improve welfare by setting a new $a_i^*(t)$ and

$a_j^*(t)$ as follows: $a_i^*(t) = a_j^*(t) = \frac{a_i(t)+a_i(t)}{2}$ as this does not change the overall effort (and therefore the chance of success) but reduces, due to the quadratic costs, the combined expected costs of the project. Therefore, an asymmetric equilibrium can never be welfare maximizing. □

A.5. Proposition 2 (Social Planner)

Applying similar methods as in Appendix A.2, we get the welfare-maximizing cumulative effort:

$$\begin{aligned}\ddot{x} &= -\frac{rn^2}{2c}h(x) + \frac{1}{2}h(x)\dot{x}^2 + r\dot{x}\\ x(0) &= 0,\\ \dot{x}_T &= \frac{n^2}{2c}h(x_T)\end{aligned}$$

Or, in terms of instantaneous effort a:

$$\dot{a} = \frac{1}{2}h(x)a^2 + ra - \frac{nr}{2c}h(x)$$

which reaches $a_T = \frac{n}{2c}h(x_T)$ at time T.

The properties derived in Lemma 3 and Lemma 4 also apply to the solution of the social planner's problem (Proposition 2).

A.6. Theorem 2 (Last-minute Rush)

To prove Theorem 2, we use continuity of x to show that the negative part of $\dot{a}$ vanishes near the deadline and is therefore strictly positive.

Proof. As $h(x(t))$ is continuous in x, it is also continuous in t. We also know from Theorem 1 that $a(t)$ is continuous in t and that it satisfies (see Equation (14))

$$\begin{aligned}\dot{a} &= -\frac{r}{2c}h(x) + \frac{2n-1}{2}h(x)a^2 + ra, & a(T) &= \frac{1}{2c}h(x(T))\\ \Leftrightarrow \dot{a} &= \frac{2n-1}{2}h(x)a^2 - r\left(\frac{1}{2c}h(x) - a\right), & a(T) &= \frac{1}{2c}h(x(T))\end{aligned}$$

As $a(t)$ and $h(x(t))$ are continuous in t.

$$\lim_{t \to T}(\frac{1}{2c}h(x(t)) - a(t)) \to (\frac{1}{2c}h(x(T)) - a(T)) = 0$$

Furthermore, as $n > \frac{1}{2}$, $h(x(t)) > 0$, $a(t) > 0$ we know that:

$$\lim_{t \to T} \frac{2n-1}{2}h(x(t))a(t)^2 \to \epsilon > 0$$

Therefore $\lim_{t \to T} \dot{a} > 0$, and thus there exists an δ in which $\dot{a}(t) > 0$ for all $[T - \delta, T]$ □

A.7. Proposition 3

Proof. (a):
This follows directly from the definition of the welfare-maximizing solution. Assume there is an equilibrium path on which the cumulative effort is higher than on the welfare-maximizing path for the first time at t_1. Then a player can unilaterally spend some ϵ effort less at time t_1 and, if the cumulative effort on the equilibrium path is ever below the welfare-maximizing path, spend ϵ more at that point in time (or never, if the effort on the equilibrium path is always higher).

Then, due to the definition of the welfare-maximizing solution, the difference in welfare is positive. As she has moved some spending from an earlier point in time to a later point in time, she is better off than all other players.

Thus, her deviation is profitable for her, and therefore this path can not be an equilibrium path.

(b):
Part 1: This is a direct result of part (a). Proof by contradiction: If there is a t': $\forall t \in [0, t']$: $a_t > a_t^*$ then $x_t > x_t^*$ $\forall t \in [0, t']$, which is a contradiction to part (a).

Part 2 (might be overtaken): From looking at Figures 2 and 3, we can see that the instantaneous welfare-maximizing effort a^* might be lower than the instantaneous equilibrium effort a at some point. In the figures, one can see the equilibrium and welfare-maximizing effort side-by-side. For $n = 2$ and $n = 3$, one can see that welfare-maximizing effort starts much higher but declines quickly towards the end. With two players, the instantaneous effort at

$t = 0$ is below 0.2 in the equilibrium, whereas it is close to 0.3 in the welfare-maximizing solution. However, toward the end, the equilibrium effort has overtaken the welfare-maximizing solution (at $t = T$). The equilibrium effort is above 0.3, whereas the welfare-maximizing solution is below 0.2.

(c):
This is easy to see from Theorem 1 and Proposition 2 that

$$a_T = \frac{1}{2c} h(x_T) \leq a_T^* = \frac{n}{2c} h(x_T^*)$$

as $x_T^* \geq x_T$, $n \geq 1$ and $h(\cdot)$ is non-decreasing. □

A.8. Free-Riding

We use the best responses from Appendix A.2 to show that

$$\frac{\partial a_i^*(\cdot)}{\partial a_{-i}(t)} \leq 0$$

everywhere.

Proof.

$$\begin{aligned}
(9) \Leftrightarrow a_i(t) &= \frac{f(x(t)) + \lambda}{2c\,(1 - F(x(t)))} \\
\Leftrightarrow \lambda &= 2ca_i(t)\,(1 - F(x(t))) - f(x(t)) \\
\Rightarrow \dot{\lambda} &= 2c\,(1 - F(x(t)))\,\dot{a}_i(t) - \dot{x}(t)\,(2ca_i(t)f(x(t)) + f'(x(t))) \\
10 \Rightarrow &\ 2c\,(1 - F(x(t)))\,\dot{a}_i(t) - 2cf(x(t))a_i(t)^2 - 2cf(x(t))a_i(t)a_{-i}(t) - f(x(t))a_i(t) - f(x(t))a_{-i}(t) \\
11 \Rightarrow \dot{\lambda} &= 2rca_i(t)\,(1 - F(x(t)))\,a_i(t) - f(x(t))r - f'(x(t))a_i(t) - f'(x(t))a_{-i}(t) - ca_i(t)^2 f(x(t)) \\
\Rightarrow &\ (1 - r)\,2c\,(1 - F(x(t)))\,a_i(t) - cf\,a_i(t)^2 = -f(x(t))r + 2cf(x(t))a_i(t)a_{-i}(t) \\
\Leftrightarrow &\ 2h(x(t))a_i(t) - a_i^2 - 2a_i(t)a_{-i}(t) = -\frac{r}{c}
\end{aligned}$$

Solving for $a_i(\cdot)$ and taking the partial derivative gives:

$$\frac{\partial a_i^*(\cdot)}{\partial a_{-i}(t)} = \frac{(a_{-i}(t) - h(x))}{\sqrt{\frac{c}{r} + (a_{-i}(t) - h(x))^2}} - 1$$

Which is always negative, as long as $c > 0$ and $(1-r)\,h(x(t)) \neq a_{-i}(t)$ and 0 otherwise. Therefore, we always have a free-riding effect. □

A.9. Theorem 3 (Encouragement Effect)

Proof. Using the best response $a_i^*(\cdot)$ from Appendix A.8 and taking the partial derivative with respect to $x(t)$ yields:

$$\begin{aligned}\frac{\partial a_i^*(\cdot)}{\partial x(t)} &= \frac{(h(x(t)) - a_{-i}(t))\,h'(x(t))}{\sqrt{\frac{r}{c} - 2a_{-i}(t)h(x(t)) + h(x(t))^2 + a_{-i}(t)^2}} + h'(x(t)) \\ &= h'(x(t))\left(\frac{h(x(t)) - a_{-i}(t)}{\sqrt{\frac{r}{c} + (h(x(t)) - a_{-i}(t))^2}} + 1\right)\end{aligned}$$

Which is, for $h'(x(t)) = 0$ (i.e., a constant hazard rate) always 0 and, if $c > 0$ and $h(x(t)) \neq a_{-i}(t)$ for $h'(x(t)) > 0$ positive and for $h'(x(t)) < 0$ always negative. □

B. Certain Objectives

First, before showing the proof for Proposition 5, let me illustrate the main difference between Georgiadis (2017) model without monitoring and this model with certain objectives with an example.

Example. *Assume that the costs are low enough, s.t. (abusing the notation of $c(\cdot)$): $c\left(\frac{\bar{x}}{n}\right) \leq 1 - e^{-rT}$. Then the losses due to delaying the reward until T are higher then the highest possible costs that can occur in the symmetric equilibrium.*

Let us compare the utility from getting the work done at $t = 0$: $V_0 = 1 - c\left(\frac{\bar{x}}{n}\right)$ and from getting the work done at $t = T$: $V_T = e^{-rT} - c_T$ where c_T are some non-negative costs. Then we know that:

$$V_0 = 1 - c\left(\frac{\bar{x}}{n}\right) \quad > \quad e^{-rT} - c_T = V_T$$

Completing the project at $t = 0$ is strictly better (but not necessarily optimal). Therefore, working until the end can never be optimal, whereas in Georgiadis (2017) it is always better to work until the deadline.

Proof: Proposition 5. The solution to the case of $r = 0$ is trivial: $\bar{x}$ is distributed evenly between $t = 0$ and T.

The second case is less obvious, but the problem can be split into two problems: one of the optimal effort allocation, which has been solved by Georgiadis (2017) and one of finding the optimal t^* to finish the project.

The solution to the first part, following Georgiadis (2017), is that the players will choose efforts such that their discounted marginal costs are constant, i.e., $a_i = \frac{r\bar{x}}{n}\frac{e^{rt}}{e^{rt^*}-1}$.

The solution to the second part is an optimization problem, in which we maximize the expected discounted playoff of the players, given that the project is profitable, i.e. $r\bar{x}^2 < 2n^2$:

$$\begin{aligned}
&\max_{t'} e^{-rt'} - \frac{r\bar{x}^2}{2n^2}\frac{1}{e^{rt'}-1} \\
\Rightarrow & t' = \frac{1}{r}\log\left(n\left(\frac{\sqrt{2}\bar{x}\sqrt{r}}{2n^2-\bar{x}^2 r} + \frac{2n}{2n^2-\bar{x}^2 r}\right)\right)
\end{aligned}$$

Finding the maximum and verifying that it is a maximum can be done by simple calculations. Furthermore, deviations from this t^* can never be profitable in the symmetric equilibrium, as one would either work earlier without moving the project's completion or is moving t^* back, as it would lower your payoff.

Now, we can see that the stopping time is $t^* = \min\{t', T\}$. □

C. The Breakthrough-Effort Distribution

The most important characteristic of a project in this model is the "breakthrough-effort distribution" or, in other words, how much effort one has to spend for a certain chance of success, given the effort that has been spent by the team in the past. Therefore, this distribution describes how likely every possible effort threshold is at the project's present stage. If the player thinks finding a cure for a disease costs around 100 billion staff-hours of research, she could assume some normal distribution around 100 billion. If I am sure I lost my keys in my apartment (again) but have no idea where they could be, assuming a uniform distribution over every room in my apartment seems reasonable.

In this section, we are looking at three classes of distributions and how they can be interpreted in the context of the model. The distributions will be denoted by their hazard rates $h(x(t)) := \frac{f(x(t))}{1-F(x(t))}$, which describes the effect of past effort on the effectiveness of current effort, or, in other words, the probability that the project will be successful now, given that it was not successful so far.

Example 1 (Constant hazard rate). *The first type of distribution has a constant hazard rate, i.e., the exponential distribution ($F(x) = 1 - e^{-\lambda x}$ with a rate parameter $\lambda > 0$). This distribution conveys the idea that the chance of success only depends on the current effort, and past effort does not matter at all. For example, if trying to roll a six on a dice: The chance of success is independent of prior dice rolls.*

Example 2 (Decreasing hazard rate). *A variation of the exponential distribution is an incomplete exponential distribution (i.e., an exponential distribution with a probability mass at infinity).[15] Although technically a distribution with a decreasing hazard rate, the intuition is similar to the memoryless distribution example: The probability distribution itself is memoryless. However, there is a chance of failure: As time proceeds, the expected probability of failure is updated and therefore increases in the effort already spent. A popular example of a decreasing hazard rate is a search model, similar to Keller, Rady, and Cripps (2005), where you search at the most likely places first or investments into R&D: the more you invest without success, the higher is your belief that there is no solution to the problem.*

Example 3 (Increasing hazard rate). *The last example is increasing hazard rates (e.g., when the breakthrough effort is distributed uniformly on some interval). Possible applications are projects with a strong learning-by-doing effect and projects where the success in a certain period depends on the cumulative effort, not on current effort.[16] For example, a legal team is trying to find a particular file in a room full of documents.*

For more examples, see Section 2 in Khan and Stinchcombe (2015), which provides an overview of the meaning of success probability distributions, their hazard rates, and their relations to different projects.

Although all examples in this paper will be from one of the three classes, the results also hold for general distributions.

[15] Using this distribution in this model yields us a model very similar to the so-called good news bandit models. One example is Bonatti and Hörner's (2011) benchmark model; the only difference being that we use quadratic instead of linear costs.

[16] One example is Georgiadis (2014). In his model, the uncertainty is about the effect of effort and not the threshold, but this is just a different way to model uncertainty about the relationship between effort spent and success. One can, therefore, generate a very similar model in the framework presented by choosing the appropriate breakthrough effort distribution, which would have an increasing hazard rate.

C.1. Stochastic project progress

A simple example how a add stochastic project progress is to add noise ε to the effectiveness of the players' effort:

$$\overline{x} \leq \int_{0}^{t'} \sum_{i=1}^{n} ((1 + \varepsilon)\, a_i(t))\, \mathrm{d}t$$

From this, one can derive a new "breakthrough-effort" distribution (analogously to Appendix A.1). As long as this distribution fulfills the conditions in Assumption 1, all results in the paper still hold.

In the example above, the "noisy" breakthrough-effort distribution fulfills the conditions if the original breakthrough-effort distribution (without stochastic project progress) fulfills them.

This is not true for general types of stochastic project progress. Even more so, every stochastic project progress which does not fulfill a "no-effort no-progress" condition does result in a continuous or bounded hazard rate and thus violates Assumption 1.

D. Constant Hazard Rate

Let us have a closer look at the case of constant hazard rates, i.e., the exponential distribution ($F(x) = 1 - e^{-\lambda x}$ with a rate parameter $\lambda > 0$). This distribution conveys the idea that the chance of success only depends on the current effort, and past effort does not matter at all. For example, if trying to roll a six on a dice: The chance of success is independent of prior dice rolls.

For the exponential distribution, the solution from Theorem 1 reduces to:

$$\begin{aligned} \dot{a} &= \frac{2n-1}{2}\lambda a^2 + ra - \frac{r}{2c}\lambda \\ a_T &= \frac{1}{2c}\lambda \end{aligned} \tag{15}$$

which has the following explicit solution:

$$a(t) = \frac{\lambda\left(e^{\frac{(t-T)\sqrt{cr(cr+\lambda^2(2n-1))}}{c}}+1\right)\left(\lambda^2(2n-1)+cr-\sqrt{cr\left(cr+\lambda^2(2n-1)\right)}\right)+2\sqrt{cr\left(cr+\lambda^2(2n-1)\right)}}{2c\left(cr+\lambda^2(2n-1)\right)\left(\left(1-\frac{2cr+\lambda^2(2n-1)}{2\sqrt{cr(cr+\lambda^2(2n-1))}}\right)e^{\frac{(t-T)\sqrt{cr(cr+\lambda^2(2n-1))}}{c}}+\frac{2cr+\lambda^2(2n-1)}{2\sqrt{cr(cr+\lambda^2(2n-1))}}+1\right)}$$

with λ being the rate parameter of the exponential distribution.

The welfare-maximizing effort has the following explicit solution

$$a(t) = \frac{\lambda n e^{-T\sqrt{r\left(\frac{\lambda^2 n}{c}+r\right)}}\left(\left(r-\sqrt{r\left(\frac{\lambda^2 n}{c}+r\right)}\right)e^{t\sqrt{r\left(\frac{\lambda^2 n}{c}+r\right)}}-\left(\sqrt{r\left(\frac{\lambda^2 n}{c}+r\right)}+r\right)e^{T\sqrt{r\left(\frac{\lambda^2 n}{c}+r\right)}}\right)}{\lambda^2 n\left(e^{(t-T)\sqrt{r\left(\frac{\lambda^2 n}{c}+r\right)}}-1\right)+2cr\left(e^{(t-T)\sqrt{r\left(\frac{\lambda^2 n}{c}+r\right)}}-1\right)-2\sqrt{cr\left(cr+\lambda^2 n\right)}\left(e^{(t-T)\sqrt{r\left(\frac{\lambda^2 n}{c}+r\right)}}+1\right)}$$

with λ being the rate parameter of the exponential distribution.

Given this solution, some observations about this class of distributions can already be made: Independent of the number of players (and the discount rate), the individual effort right before the deadline is always the same in the equilibrium. Furthermore, we can see that the individual effort decreases in the number of players, although the reward for completion for each player is independent of the number of players.

Furthermore, in this example, we can directly compare the non-cooperative equilibrium and the socially optimal effort. It turns out that in this case, the socially optimal effort is always higher than the equilibrium effort.

As we have no encouragement effect (only pure free-riding), it is clear that the effect at work here is delay of effort. Furthermore, without a deadline (i.e., $T = \infty$), the effort would be constant.[17] However, if we introduce a deadline, this changes, and we see an increasing effort.

[17] To see this, we can compare the problem at $t_0 = 0$ and any other time t: The only differences between these two problems are the past time t and the effort already exerted $x(t)$. As t in the past does not influence the best response now, the time left is the same and, due to the properties of the exponential distribution, $x(t)$ has no effect on the beliefs about the success, the problems we are facing at t_0 and t are the same. Therefore, assuming we also have a unique best response, the continuation strategies at t_0 and t are the same for every t, i.e., players exert a constant effort.

Therefore, we can see from this example that delay of effort is a direct consequence of the deadline.

D.1. Welfare Improvement due to Uncertain Objectives

While it is, in general, beneficial to know the objectives, there are examples in which an uncertain objective improves welfare. One is given in Figure 4, in which we have two patient players, and $\bar{x}$ is with 80% 0.9 and with 20% 1.1 and a deadline $T = 1$. It is, therefore, always beneficial for the team to complete the project.

Projects and team dynamics.

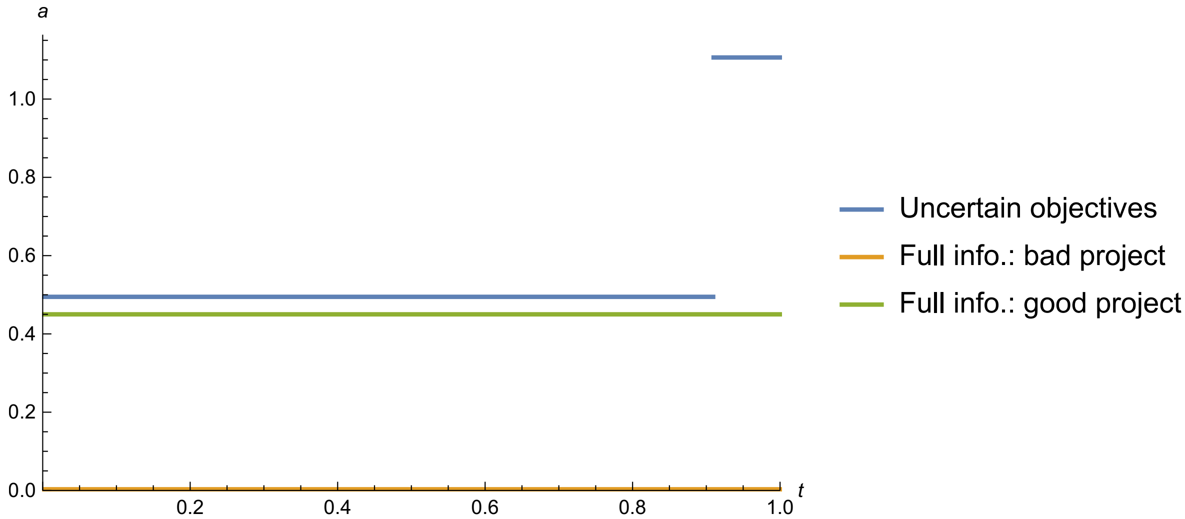

Figure 4: Full information vs uncertain objectives

However, under full information and in the “bad“ case, there are two symmetric Nash equilibria. A good one, in which both players work, and a bad one in which both players do nothing. Unfortunately, the “good“ equilibrium is not robust, as it does not fulfill the ”private feasibility” condition (see Myatt and Wallace (2008) for a related model). However, in the uncertain objective case, the only symmetric Nash equilibrium is depicted by the blue line.

E. Patient Players

So far, we have only considered the problem in which players are impatient. For patient players ($r = 0$), the solution from Theorem 1 simplifies to

$$\dot{a} = \frac{2n-1}{2} h(x) a^2$$
$$a_T = \frac{1}{2c} h(x_T)$$

Example 4 (Constant Hazard Rate)**.** *For the exponential distribution with rate parameter λ and $r = 0$, the solution from Theorem 1 reduces to:*

$$x(t) = \frac{2\lambda(\log(2c + (2n-1)T) - \log(2c + (2n-1)(T-t)))}{2n-1}$$

which is clearly increasing in t.

As we know that $h(x)$ and a are always positive, the following Proposition 6 is an obvious result:

Proposition 6. The equilibrium effort of patient players (i.e. $r = 0$) is increasing everywhere, concave for decreasing hazard rates and convex for increasing hazard rates.

It is not surprising that, without an incentive to work early, we observe even more delay of effort, i.e., effort is shifted towards the end.

F. Deadlines

We have already seen that deadlines induce delay of effort, i.e., an accumulation of effort shortly before the deadline. But is it possible to improve welfare by a deadline? Given that we only consider rational individuals, one would not expect a deadline to be beneficial if the hazard rate of the breakthrough-effort distribution is constant or even increasing. Now, Bonatti and Hörner (2011) have shown that, in their setup (i.e., with a specific type of decreasing hazard rates and linear costs), there is always a welfare-improving deadline.

However, with quadratic costs, we could not identify any situation in which deadlines improve welfare. Simulations suggest that the welfare-maximizing deadline is always the least restrictive (i.e., the deadline that allows the most time to complete the task). This is probably due to the convex costs, making it cheaper to spend effort over a longer time.

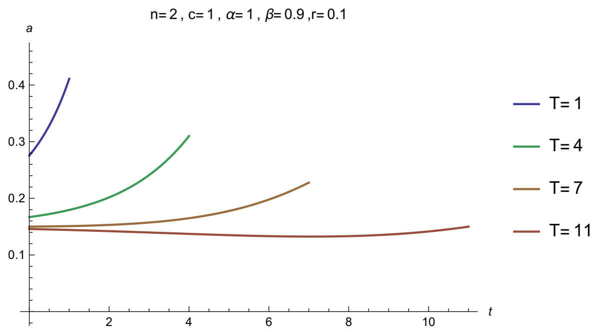

Figure 5: Effect of deadlines on efforts

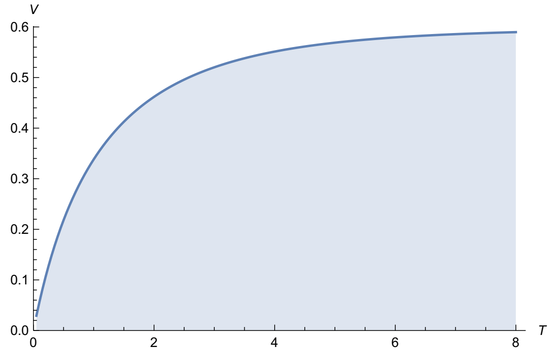

Figure 6: Effect of deadlines on welfare

In Figure 5, you can see that the efforts behave as expected: Shorter deadlines lead to overall higher efforts and, given a short enough deadline, we can even prevent the decrease of effort early on. However, the effect of a (shorter) deadline on the utility (and therefore the welfare) is, at least in simulations, always negative, as shown in the example in Figure 6.[18]

The question of whether deadlines can improve welfare is, therefore, still open. However, simulations suggest that (shorter) deadlines are never beneficial in this model.

G. The Effect of Team Size

So far, we have assumed a fixed number of players. What happens if the number of players changes?

Given that the reward from successful completion of the project is modeled as non-rivalrous, it is not surprising that the welfare increases in the number of players.

To show this, remember that, in this model, players never over-exert in the equilibrium. Let's start by $n = 1 \rightarrow n = 2$: Assume there was a 2-player equilibrium in which the players were worse off than in the $n = 1$ case. Then, each player had a profitable deviation of playing the $n = 1$ player strategy, having exactly the same costs as in the $n = 1$ player case and a strictly higher cumulative effort $x(t)$ at every point in time and therefore at least marginally higher chances of winning before time t.[19] As the same argument also works for $N \rightarrow N + 1$, we know that welfare strictly increases in the number of players.

[18]Which again uses an incomplete exponential distribution as in Bonatti and Hörner (2011).

[19]Strictly higher cumulative effort since the other player will never exert 0 effort as shown before. Strictly higher chances of winning before time t is due to Assumption 1.

But how about individual efforts? We know that there are two strategic effects at work, free-riding and the encouragement effect.

With a constant hazard rate, we only have free-riding and no encouragement effect. Thus, every individual will work less. As the encouragement effect only depends on the cumulative effort, we can also expect the same effect with a decreasing hazard rate, which we can observe in the example of an incomplete exponential distribution in Figure 7 and Figure 8. Here we can see the lower individual efforts and higher cumulative efforts for larger team sizes. For the increasing hazard rate, however, individual efforts can be higher as long as the encouragement effect is larger than free-riding.

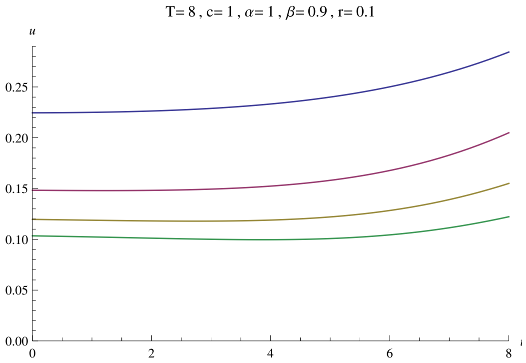

Figure 7: Individual effort for different team sizes

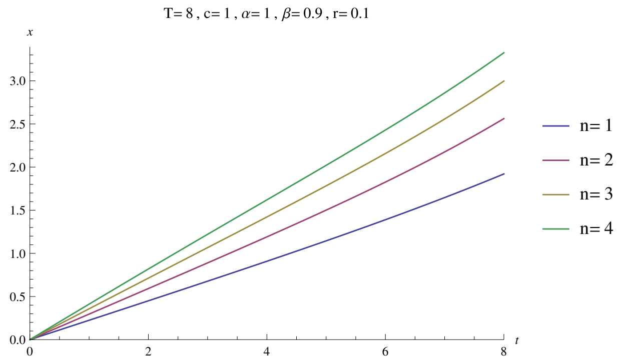

Figure 8: Cumulative effort for different team sizes

In the cases of (purely) increasing and decreasing hazard rates, the effects on individual efforts are qualitatively simple. This result, unfortunately, it does not carry over to more general hazard rates. Even for non-increasing and non-decreasing hazard rates, examples can be found under which individual efforts first increase, then decrease, and then increase again and vice versa.

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